Pattern based diving heuristics for a two-dimensional guillotine cutting stock problem with leftovers

François Clautiaux, Ruslan Sadykov, François Vanderbeck, Quentin Viaud

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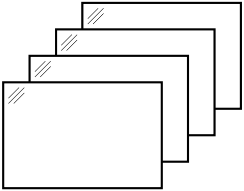
Contents

Context: production of windows

1.Initial storage



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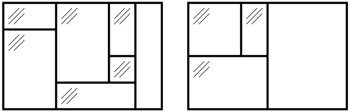
2.Cutting table



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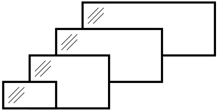
3.Intermediate storage



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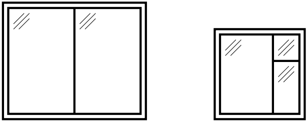
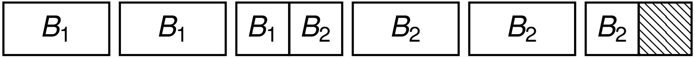
4. Assembly of windows



Specific industrial constraints

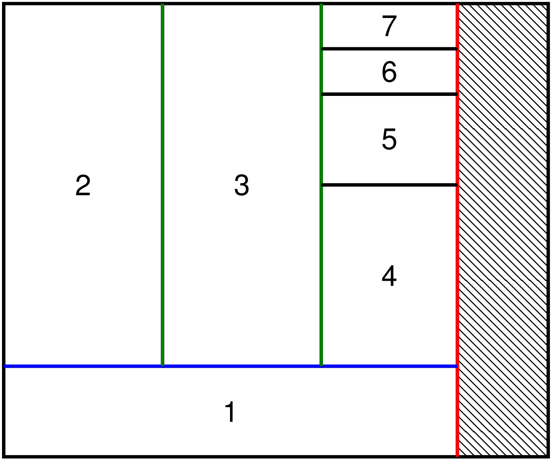
- 4-stage guillotine-cut process
- Restricted cuts (size of the cut part coincides with the width or height of a piece)
- Daily production is decomposed into independent batches (fitting to the intermediate storage)
- The order of batches is fixed because of the due dates of the customer orders
- The leftover of only the last bin of a batch can be reused for the next batch due to the organisational costs



One-batch problem : 2D guillotine cutting stock with leftovers

- Unlimited set of identical rectangular bins of size $W \times H$
- Additional leftover bin of size $\bar{W} \times H$, $\bar{W} < W$.
- Set $\mathcal{I}$ of items with fixed demand d_i (number of pieces to cut) and size $w_i \times h_i$, $i \in \mathcal{I}$
- Each item copy can be rotated by 90°
- Each item copy should be cut in at most 4 stages
- Each cut is restricted and guillotine (from one side to the opposite side)
- The objective function is to minimize the total width of used bins and the width of the used part of the last bin.

Example of a feasible cutting pattern



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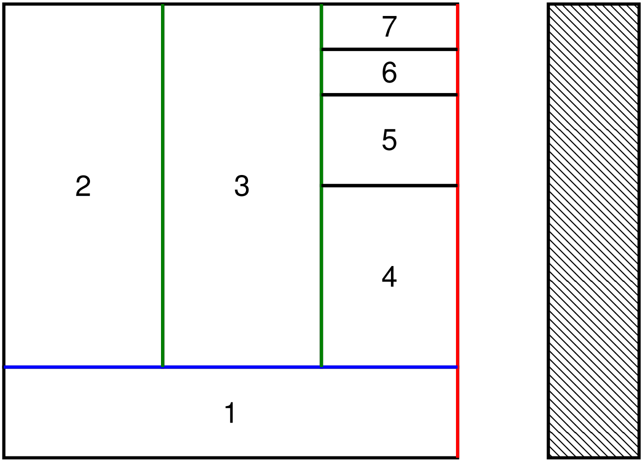

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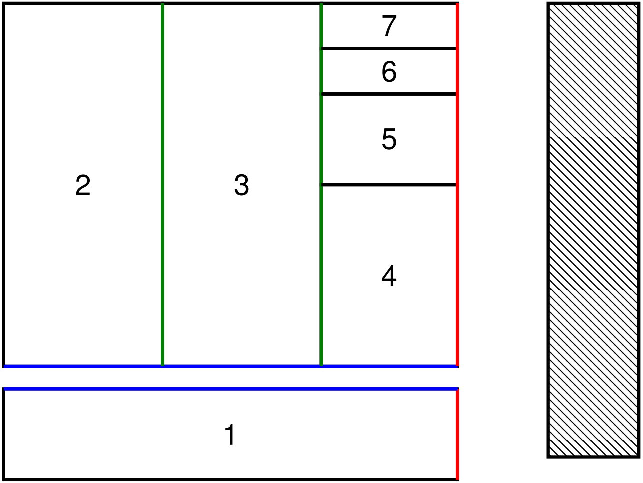

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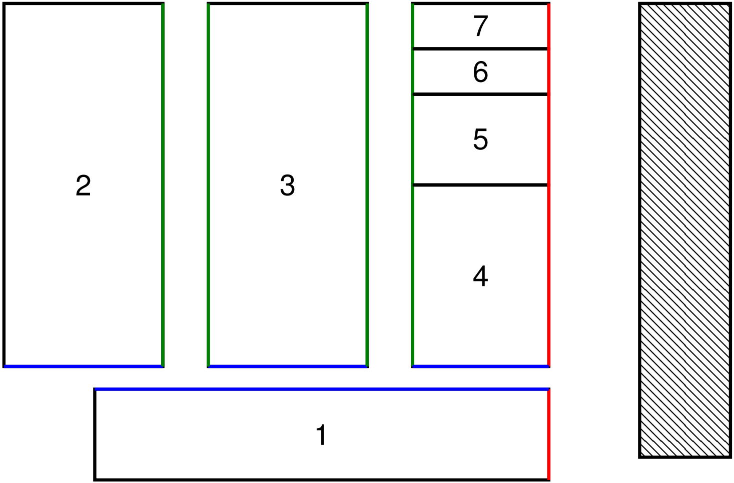

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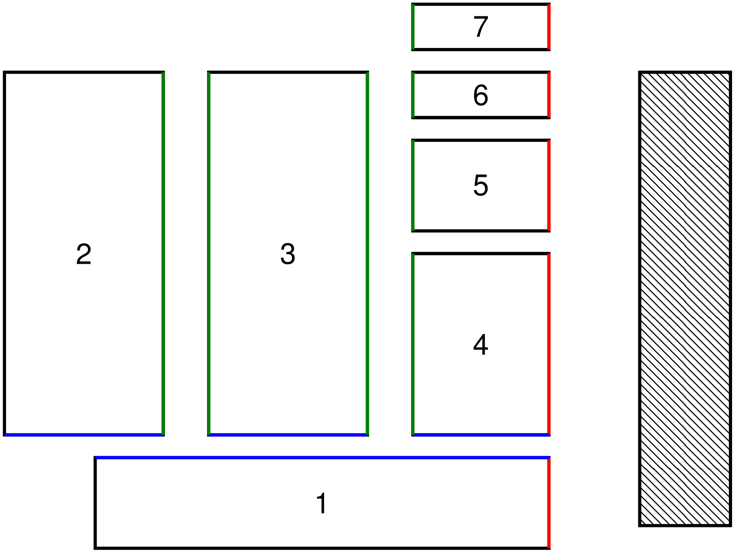
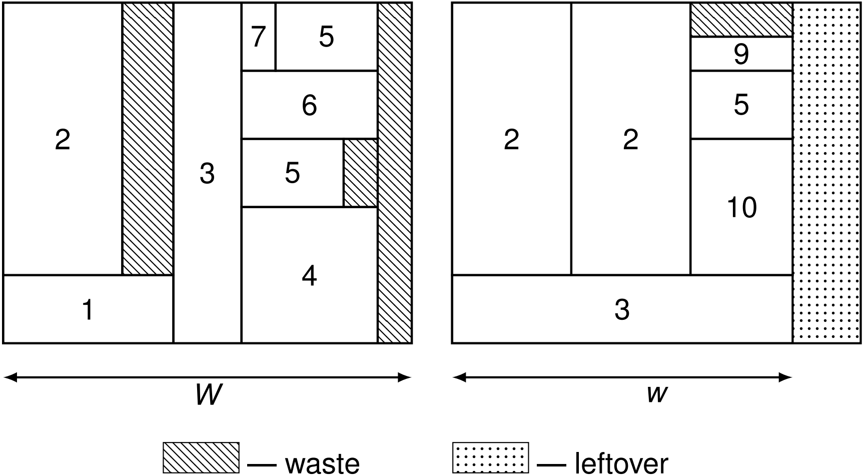


Example of a valid solution



Solution value = $W + w$

2D guillotine cutting stock : literature review

- Column generation + rounding (3 stages) [Vanderbeck, 2001]
- Branch and price (3 stages) [Puchinger and Raidl, 2007]
- Column generation + heuristics for the residual problem after the rounding [Cintra et al., 2008]
- Arc-flow MIP formulation (3 stages) [Silva et al., 2010]
- Column generation + diving (2 stages) [Furini et al., 2012]
- Dynamic MIP formulation [Furini et al., 2016]
- With leftovers (2 and 3 stages) [Puchinger et al., 2004] [Dusberger and Raidl, 2014] [Dusberger and Raidl, 2015] [Andrade et al., 2016]

Remarks

- Small instances $(W, H) = (300, 300) \Rightarrow$ Exact methods
- Large instances $(W, H) = (1000, 1000) \Rightarrow$ Heuristics
- Our one-batch instances $(W, H) = (6000, 3000)$, up to 150 items and $\approx$ 400 pieces to cut

Contents

Extended (pattern-based) formulation

- 3 bin types : leftover bin ($W' \times H$), normal bin ($W \times H$), last bin ($W \times H$)
- $\mathcal{P}_t$ — set of valid cutting patterns for a bin of type $t = 1, 2, 3$
- a_i^p — number of pieces of item i cut in pattern p
- w^p — width of pattern p

$$
\begin{aligned}
\min \sum_{p \in \mathcal{P}_1} W' \lambda_p + \sum_{p \in \mathcal{P}_2} W \lambda_p + \sum_{p \in \mathcal{P}_3} w^p \lambda_p \\
\sum_{p \in \mathcal{P}_1 \cup \mathcal{P}_2 \cup \mathcal{P}_3} a_i^p \lambda_p = d_i, \qquad & \forall i \in \mathcal{I}, \\
\sum_{p \in \mathcal{P}_t} \lambda_p = 1, \qquad & \forall t \in \{1, 3\}, \\
\lambda_p \in \mathbb{Z}_+, \qquad & \forall p \in \mathcal{P}_2, \\
\lambda_p \in \{0, 1\}, \quad & \forall p \in \mathcal{P}_t, t \in \{1, 3\}.
\end{aligned}
$$

Pricing problem

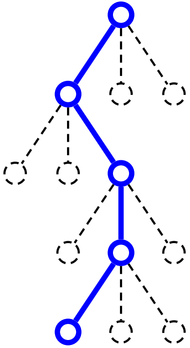
- $\pi \in \mathbb{R}^{|\mathcal{I}|}$ — dual values for the demand constraints
- $\mu = (\mu_1, \mu_3) \in \mathbb{R}^2$ — dual values for the bin number constraints
- Reduced cost of a pattern p:

$$\bar{c}_p = -\sum_{i \in \mathcal{I}} a_i^p \pi_i + \begin{cases} W' - \mu_1, & p \in \mathcal{P}_1, \\ W, & p \in \mathcal{P}_2, \\ w^p - \mu_3, & p \in \mathcal{P}_3. \end{cases}$$

- The pricing problem decomposes into three 2D guillotine integer knapsack problems, one for each bin type
- Can be solved by
  - a branch-and-bound [Puchinger and Raidl, 2007]
  - a MIP [Furini et al., 2012]
  - a dynamic program with bounds [Dolatabadi et al., 2012]
  - a labelling algorithm [Clautiaux et al., 2018]

Standard diving heuristic [Furini et al., 2012] [Sadykov et al., 2018]

- use Depth-First Search
- at each node of the tree
  - solve the master LP by column generation
  - select a pattern $p \in \mathcal{P}$ with its value $\bar{\boldsymbol{\lambda}}_{\boldsymbol{p}}$ closest to a non-zero integer $\lceil \bar{\lambda}_p \rfloor$
  - add $\lceil \bar{\lambda}_p \rfloor$ to the partial solution
  - update the master LP:
    - update demands d of the items
    - remove "non-proper" patterns p ($\exists i \in \mathcal{I} : a_i^p > d_i$)

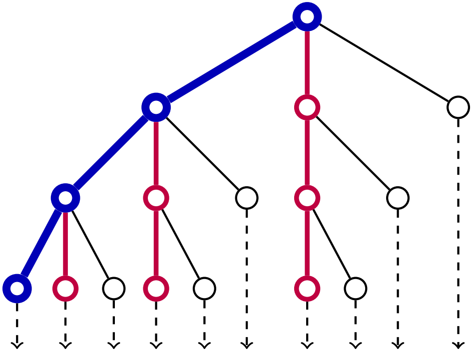


- repeat until a complete solution is obtained

The heuristic assumes that the pricing generates proper patterns ($a_i^p \leq d_i, \forall i \in \mathcal{I}$)!

Diving with LDS [Sadykov et al., 2018]

Idea: add some diversification through limited backtracking (Limited Discrepancy Search by [Harvey and Ginsberg, 1995])



MaxDiscrepancy = 2, **MaxDepth = 3**

At each node, we have a tabu list of columns forbidden to be added to the partial solution.

Contents

“Proper” vs. “non-proper” pricing

Proper case (item bounds are imposed in the pricing)

- $+$ Standard diving heuristic can be applied
- $-$ Exact pricing is expensive
- $-$ Heuristic pricing makes diving less efficient

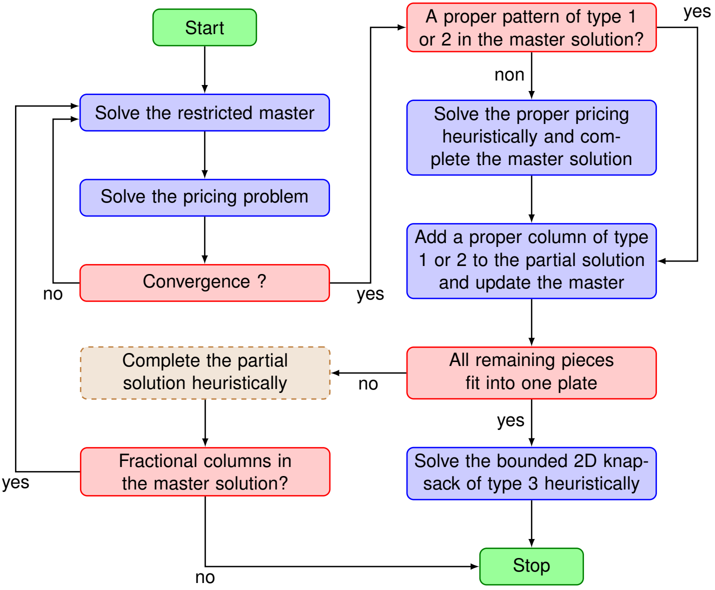
Non-proper case (unbounded pricing)

- $+$ Exact pricing by dynamic programming is relatively efficient
- $+$ Column generation dual bound is almost as tight [Cintra et al., 2008]
- $-$ One needs to adapt the diving heuristic

Diving heuristic adaptations for the “non-proper” case

- If there are not enough proper columns in the master solution, then choose ones with the smallest reduced cost
  - among all proper column in the restricted master
  - and proper columns generated with a heuristic pricing
- Never fix patterns of type 3 (for the last bin)
- When all remaining pieces fit into one bin, heuristically generate a cutting pattern minimizing its width
- Every time a partial solution is augmented, complete it heuristically (hybridization with the evolutionary heuristic)

“Non-proper” diving heuristic for our problem



Contents

Unbounded 2D guillotine knapsack: dynamic program

- Application of [Beasley, 1985] and [Russo et al., 2014]
- $\mathcal{W}(w,h)$, $\mathcal{H}(w,h)$ — set of all possible widths and heights
- $U(w,h,s)$ ($U(\overline{w,h,s})$) — max. value of a pattern of size $w \times h$ cut at stage s (next cut should cut a piece)

$$U(w,h,1) = \max\left\{0, \max_{w' \in \mathcal{W}(w,h)} \left\{U(\overline{w',h,2}) + U(w-w',h,1)\right\}\right\}$$

$$U(w,h,2) = \max\left\{0, \max_{h' \in \mathcal{H}(w,h)} \left\{U(\overline{w,h',3}) + U(w,h-h',2)\right\}\right\}$$

$$U(w,h,3) = \max\left\{0, \max_{w' \in \mathcal{W}(w,h)} \left\{U(\overline{w',h,4}) + U(w-w',h,3)\right\}\right\}$$

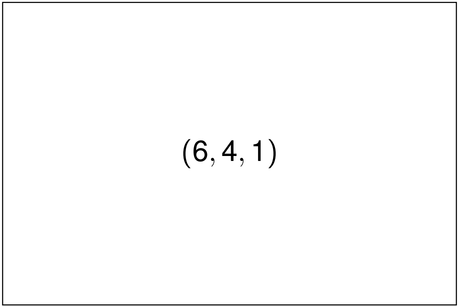
$$U(\overline{w,h,2}) = \max_{i \in \mathcal{I}:\ w_i = w, h_i \le h} \{\pi_i + U(w,h-h_i,2)\}$$

$$U(\overline{w,h,3}) = \max_{i \in \mathcal{I}:\ h_i = h, w_i \le w} \{\pi_i + U(w-w_i,h,3)\}$$

$$U(\overline{w,h,4}) = \max\left\{0, \max_{i \in \mathcal{I}:\ w_i = w, h_i \le h} \left\{\pi_i + U(\overline{w,h-h_i,4})\right\}\right\}$$

Dynamic program: example

$$U(w,h,1) = \max\{0, \max_{w' \in \mathcal{W}(w,h)} \{U(\overline{w',h,2}) + U(w-w',h,1)\}\}$$



Bin $(W,H) = (6,4)$ and an item $a = (4,3)$

Dynamic program: example

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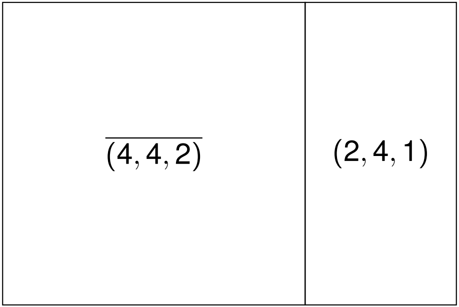

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Dynamic program: example

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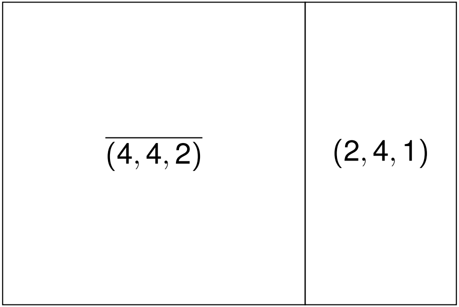

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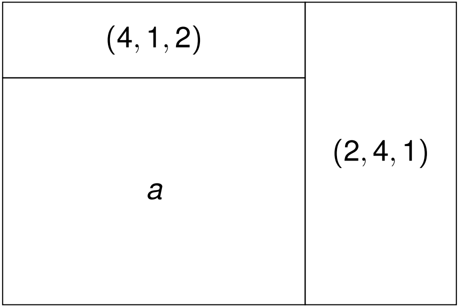


Bin $(W, H) = (6, 4)$ and an item $a = (4, 3)$

Constructive and evolutionary heuristics

Bounded 2D guillotine knapsack

- Heuristic modification of the dynamic program
  - State $U(w, h, s)$ is associated with its best partial solution
  - In the DP, we combine only the states which together satisfy the item bounds
  - Heuristic is embedded in a local search (a cut in the solution is replaced by another one)
- Evolutionary algorithm, based on [Hadjiconstantinou and Iori, 2007]
  - Every individual is a sequence of glass pieces
  - First-Fit heuristic is used to produce a complete solution
  - Two-point crossover operator

2D guillotine cutting-stock

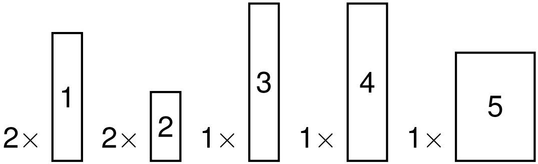
- Iteratively call above evolutionary algorithm for every bin
- List heuristics: Next-Fit, Best-Fit, First-Fit, Bottom-Fit
- List heuristics to create the vertical strips $+$ list heuristics for the 1D bin-packing

Contents

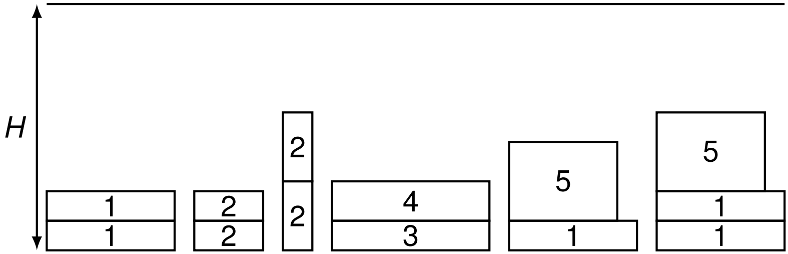
Making the pricing more "proper" : partial enumeration

- Often, only "non-proper" cutting patterns in the master solution, especially deep in the dive
  $\Rightarrow$ last fixing decisions may be bad
- Our idea is to parly take into account the item bounds and to modify accordingly the dynamic program
- Implementation is done using so-called meta-items representing stacks of item pieces satisfying item bounds
- $\mathcal{M}(w, h)$ — set of vertical meta-items containing items $i \in \mathcal{I}$, $w - \delta < w_i \leq w$, $h_i \leq h$, where $\delta = \min\limits_{i \in \mathcal{I}} w_i$,
  similar definition for set $\mathcal{M}(h, w)$ of horizontal meta-items
- Sets of meta-items are generated by enumeration
- In practice, the size of $\mathcal{M}(w, h)$ is limited

Example of meta-items



All vertical meta-items different from a single item piece



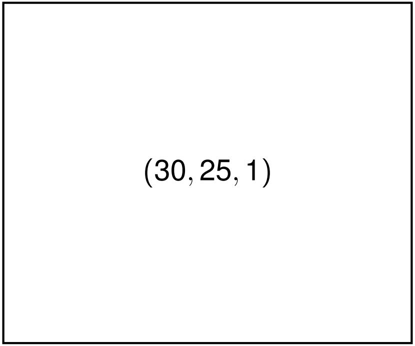
Modified dynamic program

$$U(\overline{w,h,2}) = \max_{m \in \mathcal{M}(w,h):\ \bar{h}_m \le h} \left\{ \bar{\pi}_m + U(w, h - \bar{h}_m, 2) \right\},$$

$$U(\overline{w,h,3}) = \max_{m \in \mathcal{M}(h,w-\delta):\ \bar{w}_m \le w} \{ \bar{\pi}_m + U(w - \bar{w}_m, h, 3) \},$$

$$U(\overline{w,h,4}) = \max \left\{ 0, \max_{m \in \mathcal{M}(w,h-\delta):\ \bar{w}_m = w} \{ \bar{\pi}_m \} \right\}.$$

Example



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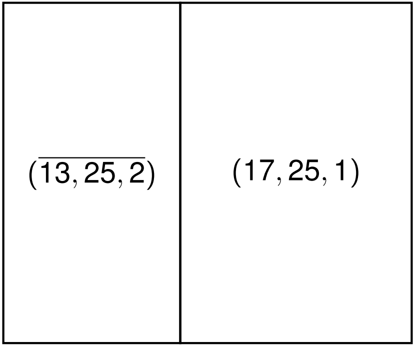

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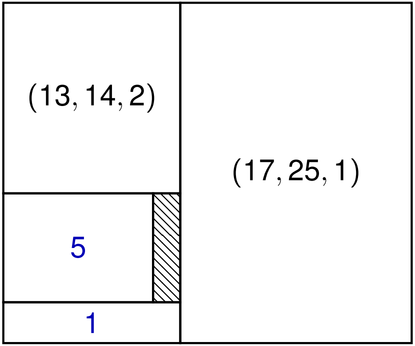

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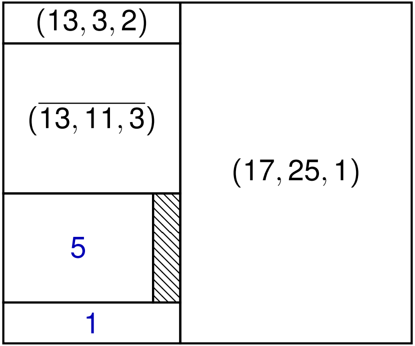

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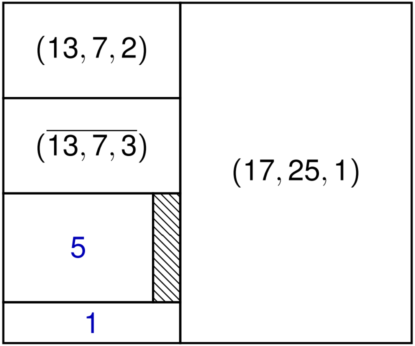

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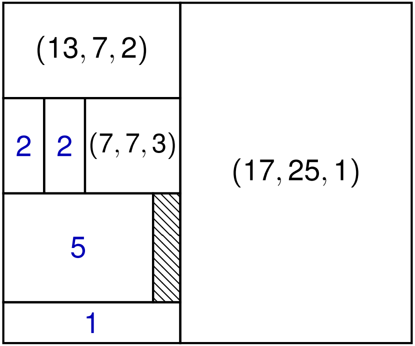
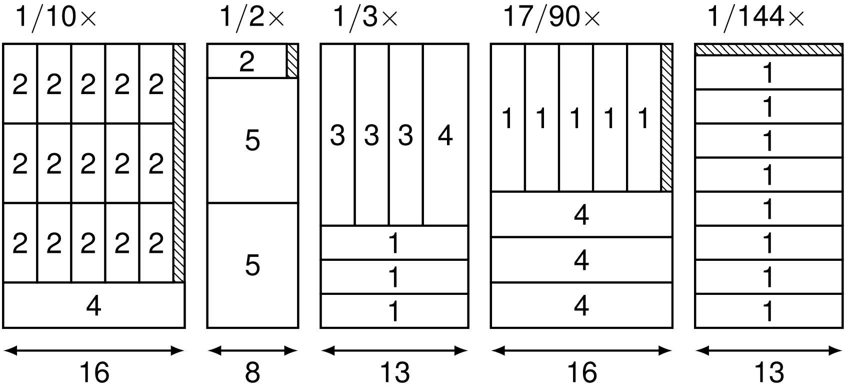


Fractional solution with standard dynamic program

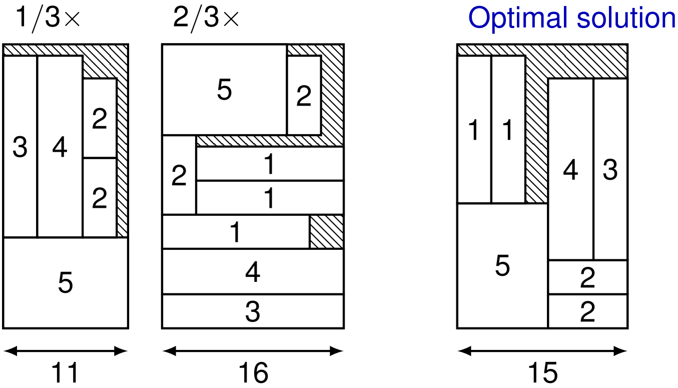
$d_1 = d_2 = 2$, $d_3 = d_4 = d_5 = 1$



Objective function : 13.046

Fractional solution with partial enumeration

$d_1 = d_2 = 2$, $d_3 = d_4 = d_5 = 1$



Objective function : 14.333

66% of the gap is closed

Contents

Instances

Academic bin-packing instances

- $\mathcal{I} \in \{20, 40, 60, 80, 100\}$
- Six classes by [Berkey and Wang, 1987]
  - $W = H \in \{10, 30, 40, 100, 300\}$
  - average piece/bin area ratio $\in \{3\%, 4\%, 20\%, 25\%\}$
- Four classes by [Martello and Vigo, 1998]
  - $W = H = 100$
  - 70% of long/wide/small/large items, 10% of each other type
  - average piece/bin area ratio $\in \{6\%, 22\%, 56\%\}$

Industrial cutting-stock instances

- $\mathcal{I} \in \{25, 50, 100\}$
- average demand ≈ 2.5
- $W \times H \in \{100 \times 50, 1000 \times 500, 6000 \times 3000\}$
- average piece/bin area ratio is 4.5%

Impact of partial enumeration technique

Dynamic program size, in thousands

	# of states		# of transitions	
Instances	stand.	p.enum.	stand.	p.enum.
Academ	2.6	3.5	25.2	29.6
Indust	8.3	10.4	123.9	162.0

Column generation gap, % from the BKS

Instances	$gap_{p.enum.}$	$gap_{stand.}$	$t_{p.enum.}$	$t_{stand.}$
Academ	1.88%	2.22%	3.9s	3.8s
Indust	1.14%	1.30%	11.2s	10.8s

Comparison with the proper bound

Instances	#conv.	t_{proper}	gap_{proper}	$gap_{p.enum.}$	$gap_{stand.}$
Academ	170/500	12m	1.29%	1.82%	2.27%
Indust	24/135	26m	0.65%	0.70%	0.90%

Comparison of heuristics (gap in % from the BKS)

- **evol** — evolutionary algorithm
- **evlist** — evolutionary + list heuristics
- **divStand** — diving with standard dynamic program
- **divPE** — diving with DPPE (dynamic program with partial enumeration)
- **divE** — hybrid diving with DPPE \ evolutionary algorithm
- **divLDS** — diving with DPPE and LDS
- **divELDS** — hybrid diving with DPPE and LDS \ evolutionary algorithm

	evol		evlist		divStand		divPE		divE		divLDS		divELDS	
Inst.	*gap*	*t*	*gap*	*t*	*gap*	*t*	*gap*	*t*	*gap*	*t*	*gap*	*t*	*gap*	*t*
Acad.	3.88	2	1.90	23	1.31	5	1.22	5	0.79	14	0.60	12	0.46	57
Indst.	2.03	4	1.92	65	1.12	20	0.75	23	0.56	59	0.38	143	0.26	448
Large	3.55	5	1.69	99	0.71	22	0.48	24	0.28	73	0.13	134	0.01	497

One-day instances (several batches)

			Solution, # glass plates				Time, minutes			
$\lvert\mathcal{B}\rvert$	$\lvert\mathcal{I}\rvert$	*LB*	evol	evlist	divPE	divE	evol	evlist	divPE	divE
10	100	123.3	126.5	126.4	124.4	124.4	1	32	24	48
10	150	191.5	195.5	195.3	192.8	192.5	4	144	83	194
15	100	191.7	196.4	196.2	193.5	193.2	2	50	36	72
15	150	277.3	283.3	282.9	279.2	279.0	6	208	126	291

- *divPE* saves up 1.4% of plates in comparison with the evolutionary heuristic
- The gap of *divPE* with the lower bound is at most 0.9%

Conclusions

- An industrial variant of the 2D guillotine cutting-stock problem is considered
- Very large practical instances (available online)
- Column generation-based "non-proper" diving heuristic is proposed
- Partial enumeration technique for the pricing DP allows us to improve the heuristic quality at virtually no cost
- Significant raw material savings due to the diving heuristic in comparison with the evolutionary one

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